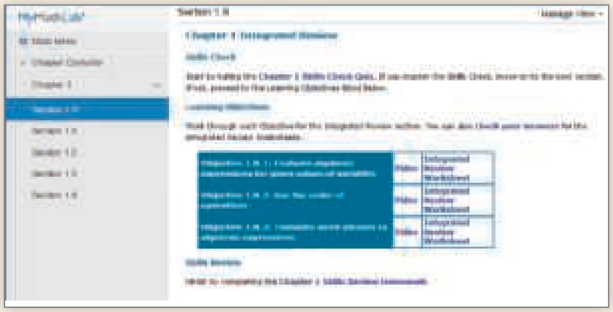
A SURVEY OF MATHEMATICS WITH APPLICATIONS

ANGEL • ABBOTT • RUNDE

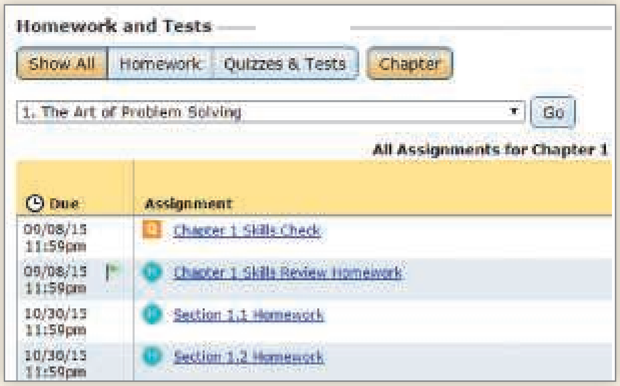
10TH EDITION

Get the Most Out of Your Liberal Arts Mathematics MyMathLab® Courses

MyMathLab with Integrated Review



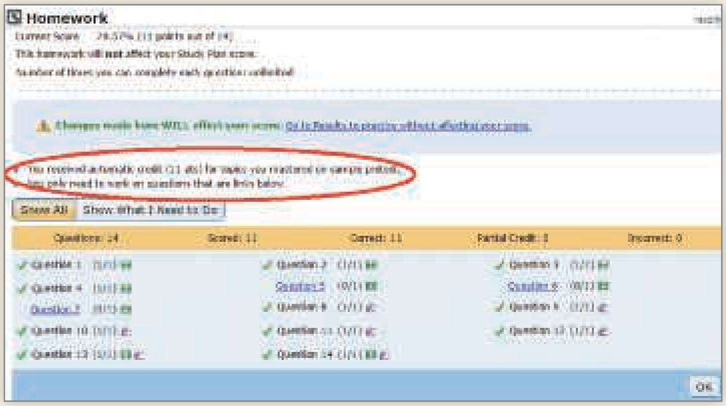
Get students caught up on topics they need to review. The Integrated Review course option in MyMathLab offers a complete liberal arts math course along with integrated review of select topics from developmental math. This solution is able to support a co-requisite course model or any course where students would benefit from review of prerequisite skills.



Students begin with a Skills Check assignment on prerequisite topics. For those who need it, developmental topic videos and worksheets are available for remediation. A personalized homework review provides extra practice to demonstrate mastery of these prerequisite skills.

Personalized Homework

MyMathLab can personalize homework assignments for students based on their performance on a test or quiz. This way, students can focus on just the topics they have not yet mastered.

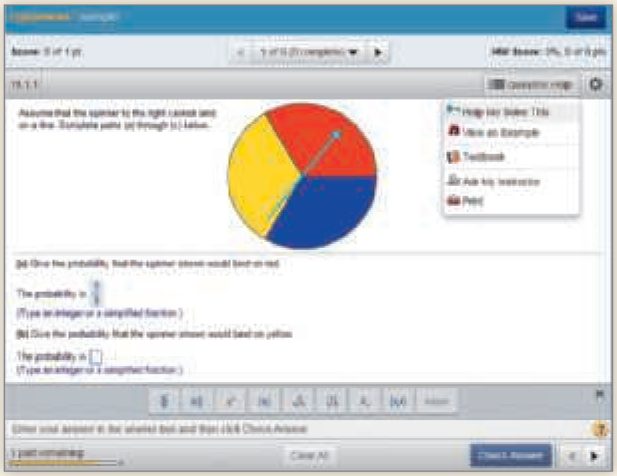


BREAK THROUGH
To improving results

Support You Need, When You Need It

Interactive Exercises

MyMathLab® interactive exercises provide immediate feedback and include learning aids such as guided solutions, sample problems, extra help at point-of-use, and helpful feedback when you enter incorrect answers.

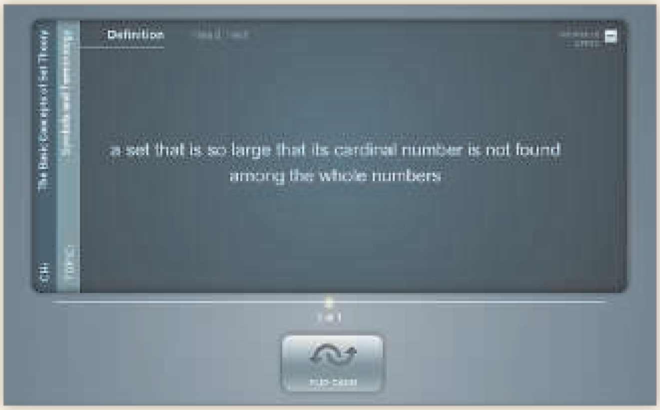


Multimedia Resources

Videos and animations in the course can help you master important concepts and terms and help you to visualize how these topics are relevant. These media assets are now accessible on your mobile device!

Flashcards

Electronic flashcards are available to enable you to review definitions, properties, and theorems in a fun online format. You can view by term or by definition and even make quizzes to help you study. Flashcards are also accessible on your mobile device so you can study whenever you have a few minutes.



10TH EDITION

A Survey of Mathematics with Applications

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To my wife, Kathy Angel
A. R. A.

To my sons, Matthew and Jake Abbott
C. D. A.

To my mother, Tina Runde, and the memory of my father, Bud Runde
D. C. R.

Contents

To the Student

Mathematics is an exciting, living study. Its applications shape the world around you and influence your everyday life. We hope that as you read this book you will realize just how important mathematics is and gain an appreciation of both its usefulness and its beauty. We also hope to teach you some practical mathematics that you can use every day and that will prepare you for further mathematics courses.

The primary purpose of this text is to provide material that you can read, understand, and enjoy. To this end, we have used straightforward language and tried to relate the mathematical concepts to everyday experiences. We have also provided many detailed examples for you to follow.

The concepts, definitions, and formulas that deserve special attention are in boxes or are set in boldface, italics, or color type. In the exercise sets, within each category, the exercises are graded, with more difficult problems appearing at the end. At the end of most exercise sets are Challenge Problems/Group Activities that contain challenging or exploratory exercises.

Be sure to read the chapter summary, work the review exercises, and take the chapter test at the end of each chapter. The answers to the odd-numbered exercises, all review exercises, and all chapter test exercises appear in the answer section in the back of the text. You should, however, use the answers only to check your work. The answers to all Recreational Mathematics exercises are provided either in the Recreational Mathematics boxes themselves or in the back of the book.

It is difficult to learn mathematics without becoming involved. To be successful, we suggest that you read the text carefully *and work each exercise in each assignment in detail*. Check with your instructor to determine which supplements are available for your use.

We welcome your suggestions and your comments. You may contact us at the following address:

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501 Boylston St., Suite 900
Boston, MA 02116

or by email at:

math@pearson.com

Subject: for Allen Angel

Good luck with your adventure in mathematics!

Allen R. Angel

Christine D. Abbott

Dennis C. Runde

Math We Use It Every Day!

We present *A Survey of Mathematics with Applications*, Tenth Edition, with the vision in mind that we use mathematics every day. In this edition, we stress how mathematics is used in our daily lives and why it is important. Our primary goal is to give students a text they can read, understand, and enjoy while learning how mathematics affects the world around them. Numerous real-life applications are used to motivate topics. A variety of interesting and useful exercises demonstrate the real-life nature of mathematics and its importance in students' lives.

The text is intended for students who require a broad-based general overview of mathematics, especially those majoring in the liberal arts, elementary education, the social sciences, business, nursing, and allied health fields. It is particularly suitable for those courses that satisfy the minimum competency requirement in mathematics for graduation or transfer.

New to This Edition

- Many chapter and section openers contain new, interesting, and motivational information and applications that illustrate the real-world nature of the material. For example, Section 2.3 introduces Venn diagrams and set operations with the experience of purchasing a laptop computer. The Chapter 3 opener indicates how logic has become important in the programming of electronic devices such as cell phones and digital cameras.
- New in the tenth edition, Learning Catalytics—a "bring your own device" student engagement, assessment, and classroom intelligence system—is available in MyMathLab, with annotations at point-of-use for instructors in the the Annotated Instructor's Edition. LC annotations provide a corresponding code for each question as it becomes relevant to integrate into the classroom. Within Learning Catalytics, simply search for the question using the code in the textbook's annotation.
- New in MyMathLab for *A Survey of Mathematics*, Tenth Edition:
  - New! Video lecture program with assignable homework questions
  - New! Interactive Concept videos with assignable homework questions
  - New! Flashcards that work on mobile devices
  - New! Study Skills module
  - Group Projects moved from the text to the MyMathLab course
- Approximately 30% of the examples and exercises in the book are new or updated to reflect current data and topics of interest to students. The authors analyzed aggregated student usage and performance data from MyMathLab for the previous edition of this text. The results of this analysis helped improve the quality and quantity of exercises in the text that matter most to instructors and students.
- Topics such as geometry, consumer mathematics, and voting have been rewritten for greater clarity.
- New Did You Know?, Mathematics Today, and Profile in Mathematics boxes have been added, while others have been updated.
- New! *Workbook including Integrated Review Worksheets* provides additional study support with objective summaries, note taking, worked out problems and additional problems for practice. Integrated Review worksheets support the Integrated Review version of the MyMathLab course.

Content Revision

In this edition, we have revised and combined certain topics to increase student understanding.

Chapter 1 "Critical Thinking Skills," includes exciting and current examples and exercises.

Chapter 2 "Sets," includes many new applications of sets pertaining to a greater variety of relevant topics, including Facebook and other social media sites.

Chapter 3 "Logic," contains many new examples and exercises. We have updated several of the puzzles, including Sudoku, Kakuro, and KenKen.

Chapter 4 "Systems of Numeration," contains many new examples and exercises that relate to the digital language used by modern devices, including smartphones, computers, and smart televisions.

Chapter 5 "Number Theory and the Real Number System," has updated information regarding the largest prime number found and the most calculated digits of pi. We updated the examples and exercise sets to reflect current economic numbers such as the national debt, the gross domestic product, population growth, and so on.

Chapter 6 In "Algebra, Graphs, and Functions," we combined *the order of operations* and *solving linear equations* into one section to emphasize the relationship between the two topics. We also moved material on *solving systems of linear equations* and *systems of linear inequalities* into Chapter 6, and removed Chapter 7. The material on *matrices* from Chapter 7 was incorporated into Chapter 9, "Mathematical Systems."

Chapter 7 "The Metric System," has many new up-to-date examples, exercises, and photographs of real-life metric use throughout the world.

Chapter 8 "Geometry," includes many updated examples and exercises, and we rewrote several topics for greater clarity.

Chapter 9 "Mathematical Systems," has many new examples and exercises. We included a section on matrices and how they can form a group under matrix addition.

Chapter 10 "Consumer Mathematics," contains updated interest rates involving a variety of loans and investments. We rewrote some of the material for greater clarity.

Chapter 11 In "Probability," we combined the topics of *empirical probability* and *theoretical probability* into one section so that students could better see the relationship, and the difference, between the two. We updated many examples and exercises that deal with real-life applications, including video games, smartphones, and social media sites.

Chapter 12 In "Statistics," we combined the topics of sampling techniques and the misuses of statistics into one section. We updated many examples and exercises involving real-life situations.

Chapter 13 "Graph Theory," involves many new applications of graphs on a variety of topics, including Facebook, Twitter, and Linkedln.

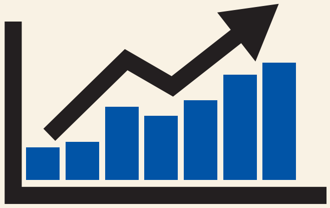
Chapter 14 "Voting and Apportionment," includes many updated examples and exercises about voting and apportionment. We rewrote some of the material for greater clarity.

Continuing and Revised Features

- **Chapter Openers** Interesting and motivational applications introduce each chapter, which includes the Why This Is Important section, and illustrate the real-world nature of the chapter topics.
- **Problem Solving** Beginning in Chapter 1, students are introduced to problem solving and critical thinking. We continue the theme of problem solving throughout the text and present special problem-solving exercises in the exercise sets.
- **Critical Thinking Skills** In addition to a focus on problem solving, this book also features sections on inductive and deductive reasoning, estimation, and dimensional analysis.
- **Profiles in Mathematics** Brief historical sketches and vignettes present stories of people who have advanced the discipline of mathematics.
- **Did You Know?** The colorful, engaging, and lively Did You Know? boxes highlight the connections of mathematics to history, the arts and sciences, technology, and a broad variety of disciplines.
- **Mathematics Today** These boxes discuss current real-life uses of the mathematical concepts in the chapter. Each box ends with Why This Is Important.
- **Recreational Math** In these boxes students are invited to apply the math in puzzles, games, and brain teasers. In addition, Recreational Mathematics problems appear in the exercise sets so that they can be assigned as homework.
- **Technology Tips** The material in these boxes explains how students can use calculators, spreadsheets, or other technologies to work certain types of application problems.
- **Timely Tips** These easy-to-identify boxes offer helpful information to make the material under discussion more understandable.
- **Boxed Material** Important definitions, formulas, and procedures are boxed, making key information easy to identify for students.
- **Chapter Summaries, Review Exercises, and Chapter Tests** The end-of-chapter summary charts provide an easy study experience by directing students to the location in the text where specific concepts are discussed. Review Exercises and Chapter Tests also help students review material and prepare for exams.

BREAKTHROUGH
To improving results

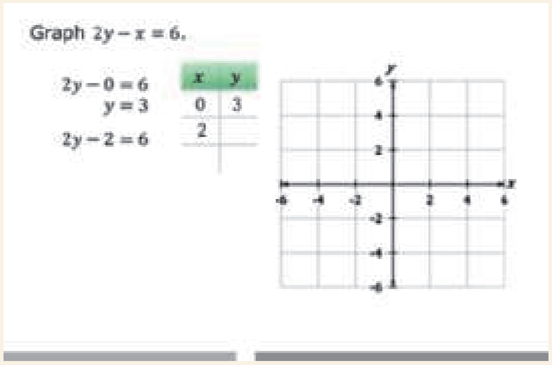
Get the most out of MyMathLab®

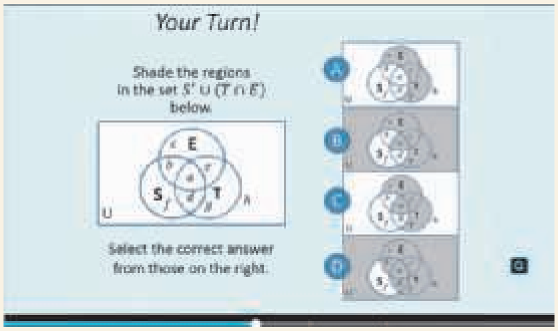


with *A Survey of Mathematics* 10e by Angel/Abbott/Runde

Videos with Assessment

NEW! A completely revised video program walks students through the concepts from every section of the text in a fresh, modern presentation format. And new video assessment questions are available, checking students' understanding of the videos they just watched and allowing the videos to be truly assignable.



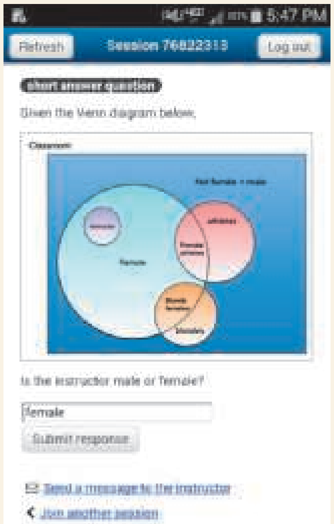


Interactive Concept Videos

NEW! Conceptual videos require students' input and interaction as they walk through a concept and pause to check understanding. Incorrect answers are followed by a video explanation of the solution and address the misconception that may have led to that particular mistake. To check students' understanding, instructors can also assign trackable exercises that correspond to the videos.

Learning Catalytics

Integrated into MyMathLab, the Learning Catalytics feature uses students' devices in the classroom for an engagement, assessment, and classroom intelligence system that gives instructors real-time feedback on student learning. To make it easy to integrate Learning Catalytics, annotations with an LC icon and code have been added to the AIE at point of use. When in Learning Catalytics, use the code to search for the specific question to use along with that topic.



Skills for Success Modules

These modules are integrated within the MyMathLab course to help students succeed in college courses and prepare for future professions.

www.mymathlab.com

BREAK THROUGH
To improving results

Resources for Success

Instructor Resources

Integrated Review MyMathLab® Course

This MyMathLab course option includes prerequisite developmental math topics—at the start of each chapter, students take a Skills Check assessment, and have the opportunity to remediate using developmental videos and Integrated Review Worksheets.

Annotated Instructor's Edition

All answers are included—when possible, answers are on the page with the exercises. Longer answers are in the back of the book. Each section includes exercises, underlined in blue, selected by the authors that can be used as homework assignments. Learning Catalytics annotations are newly included, to let instructors know when to use a relevant LC question and how to find it by searching for the code in the annotation.

The following resources can be downloaded from www.pearsonhighered.com or in MyMathLab.

TestGen®

TestGen® (www.pearsoned.com/testgen) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text.

PowerPoint® Lecture Slides

Fully editable slides correlated with the textbook are available in MyMathLab or the Instructor's Resource Center.

Instructor's Solutions Manual

This manual includes fully worked solutions to all text exercises.

Instructor's Testing Manual

This manual includes tests with answer keys for each chapter of the text.

Student Resources

Additional resources are available to support student success.

Revised Video Program

Available in MyMathLab, a completely revamped video lecture program covers every section in the text, providing students with a video tutor at home, in lab, or on the go. New interactive concept videos complete the video package, reinforcing students' conceptual understanding.

Student Solutions Manual

This manual provides detailed worked-out solutions to odd-numbered exercises, as well as to all Chapter Review and Test exercises.

Workbook including Integrated Review Worksheets

This new workbook provides objective summaries, note-taking, worked out problems, and additional practice problems. Integrated Review Worksheets support the Integrated Review MyMathLab course option and allow review and practice on prerequisite topics.

Acknowledgments

We thank our spouses, Kathy Angel and Kris Runde, for their support and encouragement throughout the project. They helped us in a great many ways, including proofreading, typing, and offering valuable suggestions. We are grateful for their wonderful support and understanding while we worked on the book.

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Finally, we thank the reviewers from all editions of the book and all the students who have offered suggestions for improving it. A list of reviewers for all editions of this book follows. Thanks to you all for helping make *A Survey of Mathematics with Applications* the most successful liberal arts mathematics textbook in the country.

Allen R. Angel

Christine D. Abbott

Dennis C. Runde

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1 Critical Thinking Skills

What You Will Learn

- Inductive and deductive reasoning processes
- Estimation
- Problem-solving techniques

Why This Is Important

Life constantly presents new problems. The more sophisticated our society becomes, the more complex the problems. We as individuals are constantly solving problems. For example, when we consider ways to reduce our expenses or when we plan a trip, we make problem-solving decisions. We also have to make problem-solving decisions when we figure out how to divide our time between studying, friends, family, work, and recreational activities. Additionally, businesses are constantly trying to solve problems that involve making a profit for the company and keeping customers satisfied.

The goal of this chapter is to help you master the skills of reasoning, estimating, and problem solving. These skills will aid you in solving problems in the remainder of this book as well as those you will encounter in everyday life.

◄ *Planning a trip uses critical thinking skills to stay within a budget while visiting exciting new places.*

SECTION 1.1 Inductive and Deductive Reasoning

Upon completion of this section, you will be able to:

- Understand and use inductive reasoning to solve problems.
- Understand and use deductive reasoning to solve problems.

Thus far, no two people have been found to have the same fingerprints or DNA. So fingerprints and DNA have become forms of identification. As technology improves, so do identification techniques. Computerized fingerprint scanners are now showing up in police stations, in high-tech security buildings, with personal computers, and with smartphones. You can purchase a personal USB fingerprint scanner that requires your distinctive fingerprint to gain access to your computer. The belief that no two people have the same fingerprints is based on a type of reasoning we will discuss in this section.

Why This Is Important As you will see in this section, we use a type of reasoning, called *inductive reasoning*, every day when we make decisions based on past experiences.

Inductive Reasoning

Before looking at some examples of inductive reasoning and problem solving, let us first review a few facts about certain numbers. The ***natural numbers*** or ***counting numbers*** are the numbers 1, 2, 3, 4, 5, 6, 7, 8, ... The three dots, called an ***ellipsis***, mean that 8 is not the last number but that the numbers continue in the same manner. A word that we sometimes use when discussing the counting numbers is "divisible." If $a \div b$ has a remainder of zero, then *a is divisible by b*. The counting numbers that are divisible by 2 are 2, 4, 6, 8, ... These numbers are called the *even counting numbers*. The counting numbers that are not divisible by 2 are 1, 3, 5, 7, 9, ... These numbers are the *odd counting numbers*. When we refer to *odd numbers* or *even numbers*, we mean odd or even counting numbers.

Recognizing patterns is sometimes helpful in solving problems, as Examples 1 and 2 illustrate.

Example 1 *The Product of Two Odd Numbers*

If two odd numbers are multiplied together, will the product always be an odd number?

Solution To answer this question, we will examine the products of several pairs of odd numbers to see if there is a pattern.

$1 \times 5 = 5$	$3 \times 7 = 21$	$5 \times 9 = 45$
$1 \times 7 = 7$	$3 \times 9 = 27$	$5 \times 11 = 55$
$1 \times 9 = 9$	$3 \times 11 = 33$	$5 \times 13 = 65$
$1 \times 11 = 11$	$3 \times 13 = 39$	$5 \times 15 = 75$

All the products are odd numbers. Thus, we might predict from these examples that the product of any two odd numbers is an odd number. ■

Example 2 *The Sum of an Odd Number and an Even Number*

If an odd number and an even number are added, will the sum be an odd number or an even number?

Solution Let's look at a few examples in which one number is odd and the other number is even.

$$3 + 4 = 7 \qquad 9 + 6 = 15 \qquad 23 + 18 = 41$$
$$5 + 12 = 17 \qquad 5 + 14 = 19 \qquad 81 + 32 = 113$$

All these sums are odd numbers. Therefore, we might predict that the sum of an odd number and an even number is an odd number. ■

In Examples 1 and 2, we cannot conclude that the results are true for all counting numbers. From the patterns developed, however, we can make predictions. This type of reasoning process, arriving at a general conclusion from specific observations or examples, is called ***inductive reasoning***, or ***induction***.

Definition: Inductive Reasoning

Inductive reasoning is the process of reasoning to a general conclusion through observations of specific cases.

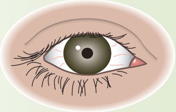
Induction often involves observing a pattern and from that pattern predicting a conclusion. Imagine an endless row of dominoes. You knock down the first, which knocks down the second, which knocks down the third, and so on. Assuming the pattern will continue uninterrupted, you conclude that any one domino that you select in the row will eventually fall, even though you may not witness the event.

Inductive reasoning is often used by mathematicians and scientists to develop theories and predict answers to complicated problems. For this reason, inductive reasoning is part of the ***scientific method***. When a scientist or mathematician makes a prediction based on specific observations, it is called a ***hypothesis*** or ***conjecture***. After looking at the products in Example 1, we might conjecture that the product of two odd numbers will be an odd number. After looking at the sums in Example 2, we might conjecture that the sum of an odd number and an even number is an odd number.

Examples 3 and 4 illustrate how we arrive at a conclusion using inductive reasoning.

MATHEMATICS TODAY

The Eyes Tell It All



Automated teller machines (ATMs) are now experimenting with determining identity by scanning the iris of a person's eye. When you open a bank account, your iris is scanned and the image is entered into a computer. When you use an ATM, a powerful camera automatically checks the veins in your iris against the computer's files. Iris scanning is also used now by law enforcement to locate missing children. In the future, iris scanning may also be used to track down Alzheimer's and mentally disabled patients. Iris scanning can be done in a matter of seconds, so it can be a much quicker procedure than fingerprinting and is just as accurate. Because the iris-scanning method of identification relies on the observation of specific cases to form a general conclusion, it is based on inductive reasoning.

Why This Is Important All of our identification procedures—fingerprints, DNA, iris scanning, etc.—are based on inductive reasoning.

Example 3 *Fingerprints and DNA*

What reasoning process has led to the conclusion that no two people have the same fingerprints or DNA? This conclusion has resulted in the use of fingerprints and DNA in courts of law as evidence to convict persons of crimes.

Solution In millions of tests, no two people have been found to have the same fingerprints or DNA. By induction, then, we believe that fingerprints and DNA provide a unique identification and can therefore be used in a court of law as evidence. Is it possible that sometime in the future two people will be found who do have exactly the same fingerprints or DNA? ■

Example 4 *Divisibility by 4*

Consider the conjecture "If the last two digits of a number form a number which is divisible by 4, then the number itself is divisible by 4." We will test several numbers to see if the conjecture appears to be true or false.

Solution Let's look at some numbers whose last two digits form a number which is divisible by 4.

Did You Know?

An Experiment Revisited

David Scott on the moon.

Apollo 15 astronaut David Scott used the moon as his laboratory to show that a heavy object (a hammer) does indeed fall at the same rate as a light object (a feather). Had Galileo dropped a hammer and feather from the Tower of Pisa, the hammer would have fallen more quickly to the ground and he still would have concluded that a heavy object falls faster than a lighter one. If it is not the object's mass that is affecting the outcome, then what is it? The answer is air resistance or friction: Earth has an atmosphere that creates friction on falling objects. The moon does not have an atmosphere; therefore, no friction is created.

Number	Do the Last Two Digits Form a Number Which is Divisible by 4?	Is the Number Divisible by 4?
324	Yes; $24 \div 4 = 6$	Yes; $324 \div 4 = 81$
4328	Yes; $28 \div 4 = 7$	Yes; $4328 \div 4 = 1082$
10,612	Yes; $12 \div 4 = 3$	Yes; $10{,}612 \div 4 = 2653$
21,104	Yes; $4 \div 4 = 1$	Yes; $21{,}104 \div 4 = 5276$

In each case, we find that if the last two digits of a number are divisible by 4, then the number itself is divisible by 4. From these examples, we might be tempted to generalize that the conjecture "If the last two digits of a number are divisible by 4, then the number itself is divisible by 4" is true.* ■

Example 5 *Pick a Number, Any Number*

Pick any number, multiply the number by 4, add 2 to the product, divide the sum by 2, and subtract 1 from the quotient. Repeat this procedure for several different numbers and then make a conjecture about the relationship between the original number and the final number.

Solution Let's go through this one together.

Pick a number:	say, 5
Multiply the number by 4:	$4 \times 5 = 20$
Add 2 to the product:	$20 + 2 = 22$
Divide the sum by 2:	$22 \div 2 = 11$
Subtract 1 from the quotient:	$11 - 1 = 10$

Note that we started with the number 5 and finished with the number 10. If you start with the number 2, you will end with the number 4. Starting with 3 would result in a final number of 6, 4 would result in 8, and so on. On the basis of these few examples, we may conjecture that when you follow the given procedure, the number you end with will always be twice the original number. ■

The result reached by inductive reasoning is often correct for the specific cases studied but not correct for all cases. History has shown that not all conclusions arrived at by inductive reasoning are correct. For example, Aristotle (384–322 B.C.) reasoned inductively that heavy objects fall at a faster rate than light objects. About 2000 years later, Galileo (1564–1642) dropped two pieces of metal—one 10 times heavier than the other—from the Leaning Tower of Pisa in Italy. He found that both hit the ground at exactly the same moment, so they must have traveled at the same rate.

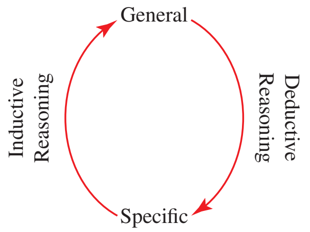
When forming a general conclusion using inductive reasoning, you should test it with several special cases to see whether the conclusion appears correct. If a special case is found that satisfies the conditions of the conjecture but produces a different result, such a case is called a ***counterexample***. A counterexample proves that the conjecture is false because only one exception is needed to show that a conjecture is not valid. Galileo's counterexample disproved Aristotle's conjecture. If a counterexample cannot be found, the conjecture is neither proven nor disproven.

Consider the statement "All birds fly." A penguin is a bird that does not fly. Therefore, a penguin is a counterexample to the statement "All birds fly."

* This statement is in fact true, as is discussed in Section 5.1.

Timely Tip

The following diagram helps explain the difference between inductive reasoning and deductive reasoning. Inductive reasoning is the process of reasoning to a general conclusion through observations of specific cases. Deductive reasoning is the process of reasoning to a specific conclusion from a general statement.



Deductive Reasoning

A second type of reasoning process is called ***deductive reasoning***, or ***deduction***. Mathematicians use deductive reasoning to *prove* conjectures true or false.

Definition: Deductive Reasoning
Deductive reasoning is the process of reasoning to a specific conclusion from a general statement.

Example 6 illustrates deductive reasoning.

Example 6 *Pick a Number, n*

Prove, using deductive reasoning, that the procedure given in Example 5 will always result in twice the original number selected.

Solution To use deductive reasoning, we begin with the *general* case rather than specific examples. In Example 5, specific cases were used. Let's select the letter n to represent *any number*.

Pick any number:	n
Multiply the number by 4:	$4n$ 4n means 4 times n.
Add 2 to the product:	$4n + 2$
Divide the sum by 2:	$\dfrac{4n+2}{2} = \dfrac{\cancel{4}^{2}n}{\cancel{2}_{1}} + \dfrac{\cancel{2}^{1}}{\cancel{2}_{1}} = 2n + 1$
Subtract 1 from the quotient:	$2n + 1 - 1 = 2n$

Note that for any number n selected, the result is $2n$, or twice the original number selected. Since n represented any number, we are beginning with the general case. Thus this is deductive reasoning. ■

In Example 5, you may have *conjectured*, using specific examples and inductive reasoning, that the result would be twice the original number selected. In Example 6, we *proved*, using deductive reasoning, that the result will always be twice the original number selected.

SECTION 1.1 *Exercises*

Warm Up Exercises

In Exercises 1–8, fill in the blank with an appropriate word, phrase, or symbol(s).

1. Another name for the counting numbers is the ________ numbers.

2. If $a \div b$ has a remainder of 0, then a is ________ by b.

3. A specific case that satisfies the conditions of a conjecture but shows the conjecture is false is called a ________.

4. A belief based on specific observations that has not been proven or disproven is called a conjecture or ________.

5. The process of reasoning to a general conclusion through observation of specific cases is called ________ reasoning.

6. The process of reasoning to a specific conclusion from a general statement is called ________ reasoning.

7. The type of reasoning used to prove a conjecture is called ________ reasoning.

8. The type of reasoning generally used to arrive at a conjecture is called ________ reasoning.

Practice the Skills

In Exercises 9–12, use inductive reasoning to predict the next line in the pattern.

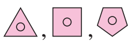
9. $1 \times 3 = 3$
$2 \times 3 = 6$
$3 \times 3 = 9$
$4 \times 3 = 12$

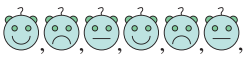
10. $15 \times 10 = 150$
$16 \times 10 = 160$
$17 \times 10 = 170$
$18 \times 10 = 180$

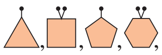
11.
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

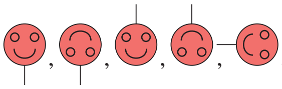
12. $10 = 10^1$
$100 = 10^2$
$1000 = 10^3$
$10{,}000 = 10^4$

In Exercises 13–16, draw the next figure in the pattern (or sequence).

13.  , . . .

14.  , . . .

15.  , . . .

16.  , . . .

In Exercises 17–26, use inductive reasoning to predict the next three numbers in the pattern (or sequence).

17. 1, 3, 5, 7, …

18. 4, 7, 10, 13, …

19. 5, -5, 5, -5, …

20. 7, 5, 3, 1, …

21. $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

22. 5, -25, 125, -625, …

23. 1, 4, 9, 16, 25, …

24. 0, 1, 3, 6, 10, 15, …

25. 1, 1, 2, 3, 5, 8, 13, 21, …

26. $2, -\frac{8}{3}, \frac{32}{9}, -\frac{128}{27}, \ldots$

Problem Solving

27. Find the letter that is the 118th entry in the following sequence. Explain how you determined your answer.

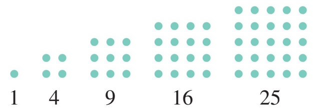
Y, R, R, Y, R, R, Y, R, R, Y, R, R, Y, R, R, …

28. a) Select a variety of one- and two-digit numbers between 1 and 99 and multiply each by 9. Record your results.

b) Find the sum of the digits in each of your products in part (a). If the sum is not a one-digit number, find the sum of the digits of the resulting sum again until you obtain a one-digit number.

c) Make a conjecture about the sum of the digits when a one- or two-digit number is multiplied by 9.

29. ***A Square Pattern*** The ancient Greeks labeled certain numbers as **square numbers**. The numbers 1, 4, 9, 16, 25, and so on are square numbers.

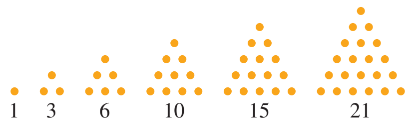


a) Determine the next three square numbers.

b) Describe a procedure to determine the next five square numbers without drawing the figures.

c) Is 72 a square number? Explain how you determined your answer.

30. ***A Triangular Pattern*** The ancient Greeks labeled certain numbers as ***triangular numbers***. The numbers 1, 3, 6, 10, 15, 21, and so on are triangular numbers.

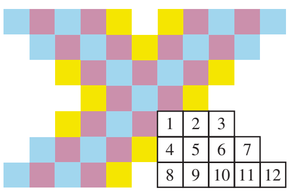


a) Can you determine the next two triangular numbers?

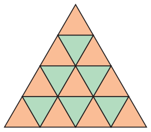
b) Describe a procedure to determine the next five triangular numbers without drawing the figures.

c) Is 72 a triangular number? Explain how you determined your answer.

31. ***Quilt Design*** The pattern shown is taken from a quilt design known as a triple Irish chain. Complete the color pattern by indicating the color assigned to each square.



32. ***Triangles in a Triangle*** Four rows of a triangular figure are shown.



a) If you added six additional rows to the bottom of this triangle, using the same pattern displayed, how many triangles would appear in the 10th row?

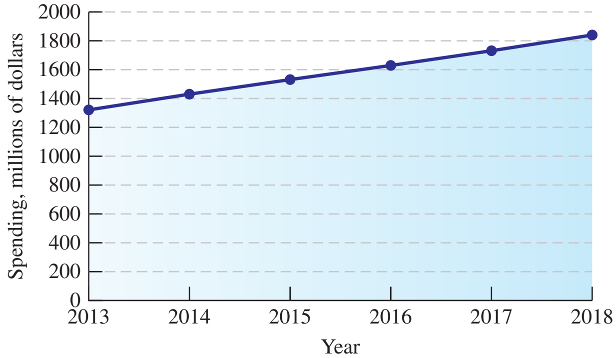
b) If the triangles in all 10 rows were added, how many triangles would appear in the entire figure?

33. ***Video Games*** The graph below shows the amount of money spent in the United States on video games played on mobile devices in 2013 and the projected amount for the years 2014–2018.

a) Assuming this trend continues, use the graph to predict the amount spent in the United States on video games played on mobile devices in 2020.

b) Explain how you are using inductive reasoning to determine your answer.

U.S. Spending on Video Games on Mobile Devices



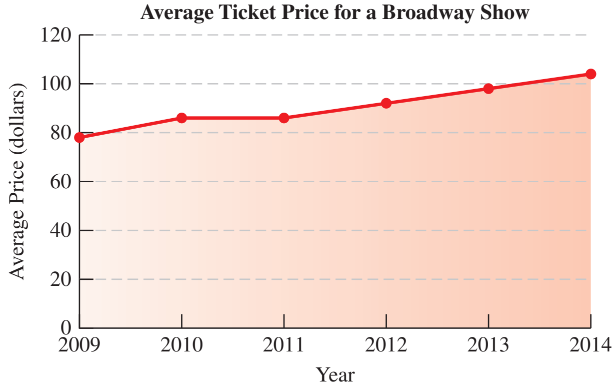
Source: Pricewaterhouse Coopers

34. ***Broadway Tickets*** The graph below shows the average ticket price for a show on Broadway for each season from 2009 through 2014.

a) Assuming this trend continues, use the graph to predict the average ticket price for a show on Broadway for the 2015 season.

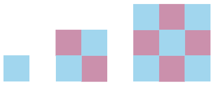
b) Explain how you are using inductive reasoning to determine your answer.

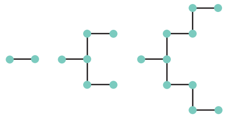
Average Ticket Price for a Broadway Show



Source: Broadway League

In Exercises 35 and 36, draw the next diagram in the pattern (or sequence).

35. 

36. 

37. Pick any number, multiply the number by 3, add 6 to the product, divide the sum by 3, and subtract 2 from the quotient. See Example 5.

a) What is the relationship between the number you started with and the final number?

b) Arbitrarily select some different numbers and repeat the process, recording the original number and the result.

c) Can you make a conjecture about the relationship between the original number and the final number?

d) Prove, using deductive reasoning, the conjecture you made in part (c). See Example 6.

38. Pick any number and multiply the number by 4. Add 6 to the product. Divide the sum by 2 and subtract 3 from the quotient.

a) What is the relationship between the number you started with and the final answer?

b) Arbitrarily select some different numbers and repeat the process, recording the original number and the results.

c) Can you make a conjecture about the relationship between the original number and the final number?

d) Prove, using deductive reasoning, the conjecture you made in part (c).

39. Pick any number and add 1 to it. Find the sum of the new number and the original number. Add 9 to the sum. Divide the new sum by 2 and subtract the original number from the quotient.

a) What is the final number?

b) Arbitrarily select some different numbers and repeat the process. Record the results.

c) Can you make a conjecture about the final number?

d) Prove, using deductive reasoning, the conjecture you made in part (c).

40. Pick any number and add 10 to the number. Divide the sum by 5. Multiply the quotient by 5. Subtract 10 from the product. Then subtract your original number.

a) What is the result?

b) Arbitrarily select some different numbers and repeat the process, recording the original number and the result.

c) Can you make a conjecture regarding the result when this process is followed?

d) Prove, using deductive reasoning, the conjecture you made in part (c).

In Exercises 41–46, find a counterexample to show that each statement is incorrect.

41. The product of any two counting numbers is divisible by 2.

42. The sum of any three two-digit numbers is a three-digit number.

43. When a counting number is added to 3 and the sum is divided by 2, the quotient will be an even number.

44. The product of any two three-digit numbers is a five-digit number.

45. The difference of any two counting numbers will be a counting number.

46. The sum of any two odd numbers is divisible by 4.

47. ***Interior Angles of a Triangle***

a) Construct a triangle and measure the three interior angles with a protractor. What is the sum of the measures?

b) Construct three other triangles, measure the angles, and record the sums. Are your answers the same?

c) Make a conjecture about the sum of the measures of the three interior angles of a triangle.

48. ***Interior Angles of a Quadrilateral***

a) Construct a quadrilateral (a four-sided figure) and measure the four interior angles with a protractor. What is the sum of the measures?

b) Construct three other quadrilaterals, measure the angles, and record the sums. Are your answers the same?

c) Make a conjecture about the sum of the measures of the four interior angles of a quadrilateral.

Concept/Writing Exercises

49. While logging on to your computer, you type in your username followed by what you believe is your password. The computer indicates that a mistake has been made and asks you to try again. You retype your username and the same password. Again, the computer indicates a mistake has been made. You decide not to try again, reasoning you will get the same error message from the computer. What type of reasoning did you use? Explain.

50. You have purchased one lottery ticket each week for many months and have not won more than $5.00. You decide, based on your past experience, that you are not going to win the grand prize and so you stop playing the lottery. What type of reasoning did you use? Explain.

Challenge Problems/Group Activities

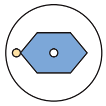
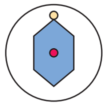
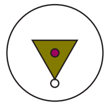
51. Complete the following square of numbers. Explain how you determined your answer.

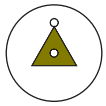
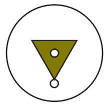
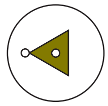
1	2	3	4
2	5	10	17
3	10	25	52
4	17	52	?

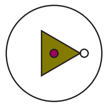
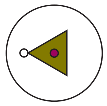
52. Find the next three numbers in the sequence.

1, 8, 11, 88, 101, 111, 181, 808, 818, 888, 1001, 1111, . . .

Recreational Mathematics

53.  is to  as  is to

a)  **b)**  **c)** 

d)  **e)** 

Research Activities

54. a) Using newspapers, the Internet, magazines, and other sources, provide examples of conclusions arrived at by inductive reasoning.

b) Explain how inductive reasoning was used in arriving at the conclusion.

55. When a jury decides whether or not a defendant is guilty, do the jurors collectively use primarily inductive reasoning, deductive reasoning, or an equal amount of each? Write a brief report supporting your answer.

SECTION 1.2 Estimation

Upon completion of this section, you will be able to:

- Use estimation techniques to determine an approximate answer to a question.

What is the approximate sales tax on a $10,000 motorcycle if the sales tax rate is 8%? What is the approximate cost to purchase 52 forty-nine-cent stamps? In this section, we will introduce a technique for arriving at an approximate answer to a question.

Why ***This Is Important*** Estimation is a technique we use every day for determining whether an answer is reasonable.

Estimation Techniques

An important step in solving mathematical problems—or, in fact, *any* problem—is to make sure that the answer you've arrived at makes sense. One technique for determining whether an answer is reasonable is to estimate. *Estimation* is the process of arriving at an approximate answer to a question. This section demonstrates several estimation methods.

To estimate, or approximate, an answer, we often round numbers, as illustrated in the following examples. The symbol $\approx$ means *is approximately equal to*.

Example 1 *Estimating the Cost of Cupcakes*

Sonya decides to purchase cupcakes for a party. Estimate her cost if she purchases 19 cupcakes at $1.95 each.

Solution We may round the amounts as follows to obtain an estimate.

Number	Number Rounded
19	20
$\times$ $1.95	$\times$ $2.00
	$40.00

Thus, the 19 cupcakes would cost approximately $40.00, written $\approx$ $40.00. ■

In Example 1, the true cost is $1.95 $\times$ 19, or $37.05. *Estimates are not meant to give exact values for answers but are a means of determining whether your answer is reasonable.* If you calculated an answer of $37.05 and then did a quick estimate to check it, you would know that the answer is reasonable because it is close to your estimated answer.

Example 2 *Two Ways to Estimate*

At a local discount retail store, Christy purchased a rug for $15.89, toothpaste for $1.49, diapers for $19.77, shampoo for $4.93, dog food for $12.88, and candy for $0.81. The cashier said the total bill was $69.51. Use estimation to determine whether this amount is reasonable.

Solution The most expensive item is $19.77 and the least expensive is $0.81. How should we estimate? We will estimate two different ways. First, we will round the cost of each item to the nearest 10 cents. For the second method, we will round the cost of each item to the nearest dollar. Rounding to the nearest 10 cents is more accurate. To determine whether the total bill is reasonable, however, we may need to round only to the nearest dollar.